Symbole (oznaczone) granic:

$$\left[\frac{a}{\pm\infty}\right] = 0,\ \ a \in \mathbb{R}.$$
$$[+\infty + (+\infty)] = +\infty, \quad \left[\frac{1}{0^+}\right] = +\infty.$$
$$[-\infty + (-\infty)] = -\infty, \quad \left[\frac{1}{0^-}\right] = -\infty.$$

1. Obliczyć $\lim\limits_{n\to\infty} \dfrac{5}{n^2-3n+2}$.
   Wyznaczymy symbol granicy. Zauważmy, że $\lim\limits_{n\to\infty}\left(n^2-3n+2\right) = +\infty$ (patrz wykres funkcji $f(x) = x^2-3x+2$).
   Zatem symbol granicy z zadania to $\left[\dfrac{5}{+\infty}\right]$, co w granicy daje wynik 0. Piszemy:
$$\lim_{n\to\infty} \frac{5}{n^2-3n+2} \overset{\left[\frac{5}{+\infty}\right]}{=} 0.$$
   Podobnie
2.
$$\lim_{n\to\infty} \frac{13}{7+2n+5n^2-4n^3} \overset{\left[\frac{13}{-\infty}\right]}{=} 0.$$
3. Obliczyć $\lim\limits_{n\to\infty}\left(3n+\sqrt{9n^2+2n-5}\right)$. Mamy
$$\lim_{n\to\infty}\left(3n+\sqrt{9n^2+2n-5}\right) \overset{[+\infty+(+\infty)]}{=} +\infty.$$
   Podobnie
4.
$$\lim_{n\to\infty}\left(\sqrt{3n^2+2n-5}+\sqrt{4n^2+5n-7}\right) \overset{[+\infty+(+\infty)]}{=} +\infty.$$

Symbole nieoznaczone granic:
$$\left[\frac{\infty}{\infty}\right].$$
$$[+\infty-(+\infty)],\ \ [-\infty-(-\infty)].$$
$$[1^\infty].$$

5. Obliczyć $\lim\limits_{n\to\infty} \dfrac{5n^2+7n-6}{4n^2-3n+2}$. Mamy
$$\lim_{n\to\infty} \frac{5n^2+7n-6}{4n^2-3n+2} \overset{\left[\frac{\pm\infty}{+\infty}\right]}{=} \left| \begin{array}{c} \text{dzielimy licznik i mianownik przez} \\ \text{"największe" wyrażenie z } \textbf{mianownika} \\ \text{(w naszym wypadku przez } n^2\text{)} \end{array} \right| = \lim_{n\to\infty} \frac{\frac{5n^2+7n-6}{n^2}}{\frac{4n^2-3n+2}{n^2}} =$$
$$= \lim_{n\to\infty} \frac{\frac{5n^2}{n^2}+\frac{7n}{n^2}-\frac{6}{n^2}}{\frac{4n^2}{n^2}-\frac{3n}{n^2}+\frac{2}{n^2}} = \lim_{n\to\infty} \frac{5+\frac{7}{n}-\frac{6}{n^2}}{4-\frac{3}{n}+\frac{2}{n^2}}.$$
   Zauważmy, że
$$\lim_{n\to\infty}\frac{7}{n} \overset{\left[\frac{7}{+\infty}\right]}{=} 0;\ \ \lim_{n\to\infty}\frac{6}{n^2} \overset{\left[\frac{6}{+\infty}\right]}{=} 0;\ \ \lim_{n\to\infty}\frac{3}{n} \overset{\left[\frac{3}{+\infty}\right]}{=} 0;\ \ \lim_{n\to\infty}\frac{2}{n^2} \overset{\left[\frac{2}{+\infty}\right]}{=} 0.$$
   Więc
$$\lim_{n\to\infty} \frac{5+\frac{7}{n}-\frac{6}{n^2}}{4-\frac{3}{n}+\frac{2}{n^2}} = \lim_{n\to\infty}\frac{5+0-0}{4-0+0} = \lim_{n\to\infty}\frac{5}{4} = \frac{5}{4}.$$
   Ostatecznie
$$\lim_{n\to\infty} \frac{5n^2+7n-6}{4n^2-3n+2} = \frac{5}{4}.$$
6. Obliczyć $\lim\limits_{n\to\infty} \dfrac{6+2n+3n^2-5n^3}{7n^2+3n-5}$. Mamy
$$\lim_{n\to\infty} \frac{6+2n+3n^2-5n^3}{7n^2+3n-5} \overset{\left[\frac{-\infty}{+\infty}\right]}{=} \left| \begin{array}{c} \text{dzielimy licznik i mianownik przez} \\ \text{"największe" wyrażenie z } \textbf{mianownika} \\ \text{(w naszym wypadku przez } n^2\text{)} \end{array} \right| = \lim_{n\to\infty} \frac{\frac{6}{n^2}+\frac{2}{n}+3-5n}{7+\frac{3}{n}-\frac{5}{n^2}} =$$
$$\lim_{n\to\infty} \frac{3-5n}{7} \overset{\left[\frac{-\infty}{7}\right]}{=} -\infty.$$

7. Obliczyć $\lim\limits_{n\to\infty}\dfrac{\sqrt{3n^2-5n+6}}{2n-5}$. Mamy

$$\lim_{n\to\infty}\frac{\sqrt{3n^2-5n+6}}{2n-5}\overset{[\frac{+\infty}{+\infty}]}{=}\left|\begin{array}{c}\text{dzielimy licznik i mianownik przez}\\ \text{"największe" wyrażenie z }\textbf{mianownika}\\ \text{(w naszym wypadku przez } n)\end{array}\right|=\lim_{n\to\infty}\frac{\frac{\sqrt{3n^2-5n+6}}{n}}{\frac{2n-5}{n}}=$$
$$\lim_{n\to\infty}\frac{\sqrt{\frac{3n^2-5n+6}{n^2}}}{\frac{2n-5}{n}}=\lim_{n\to\infty}\frac{\sqrt{3-\frac{5}{n}+\frac{6}{n^2}}}{2-\frac{5}{n}}=\lim_{n\to\infty}\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}.$$

8. Obliczyć $\lim\limits_{n\to\infty}\dfrac{3n-16}{\sqrt{n^4-5n^3+6n^2+1}}$. Mamy

$$\lim_{n\to\infty}\frac{3n-16}{\sqrt{n^4-5n^3+6n^2+1}}\overset{[\frac{\pm\infty}{+\infty}]}{=}\left|\begin{array}{c}\text{dzielimy licznik i mianownik przez}\\ \text{"największe" wyrażenie z }\textbf{mianownika}\\ \text{(w naszym wypadku przez } n^2\text{ (!!!))}\end{array}\right|=\lim_{n\to\infty}\frac{\frac{3n-16}{n^2}}{\frac{\sqrt{n^4-5n^3+6n^2+1}}{n^2}}=$$
$$\lim_{n\to\infty}\frac{\frac{3n-16}{n^2}}{\sqrt{\frac{n^4-5n^3+6n^2+1}{n^4}}}=\lim_{n\to\infty}\frac{\frac{3}{n}-\frac{16}{n^2}}{\sqrt{1-\frac{5}{n}+\frac{6}{n^2}+\frac{1}{n^3}}}=\lim_{n\to\infty}\frac{0}{\sqrt{1}}=\operatorname{arctg}0.$$

9. Obliczyć $\lim\limits_{n\to\infty}\left(3n-\sqrt{9n^2+2n-5}\right)$. Mamy

$$\lim_{n\to\infty}\left(3n-\sqrt{9n^2+2n-5}\right)\overset{[+\infty-(+\infty)]}{=}\left|\begin{array}{c}\text{zamieniamy różnicę na iloraz}\\ \text{za wzoru } a-b=\dfrac{a^2-b^2}{a+b}\end{array}\right|=\lim_{n\to\infty}\frac{9n^2-\left(9n^2+2n-5\right)}{3n+\sqrt{9n^2+2n-5}}=$$
$$\lim_{n\to\infty}\frac{-2n+5}{3n+\sqrt{9n^2+2n-5}}\overset{[\frac{-\infty}{+\infty}]}{=}\left|\begin{array}{c}\text{dzielimy licznik i mianownik przez}\\ \text{"największe" wyrażenie z }\textbf{mianownika}\\ \text{(w naszym wypadku przez } n\text{ (!!!))}\end{array}\right|=\lim_{n\to\infty}\frac{-2+\frac{5}{n}}{3+\sqrt{9+\frac{2}{n}-\frac{5}{n^2}}}=-\frac{2}{3}.$$

10. Obliczyć $\lim\limits_{n\to\infty}\left(\sqrt{3n^2+2n-5}-\sqrt{4n^2+5n-7}\right)$. Mamy

$$\lim_{n\to\infty}\left(\sqrt{3n^2+2n-5}-\sqrt{4n^2+5n-7}\right)\overset{[+\infty-(+\infty)]}{=}\left|\begin{array}{c}\text{zamieniamy różnicę na iloraz}\\ \text{za wzoru } a-b=\dfrac{a^2-b^2}{a+b}\end{array}\right|=\lim_{n\to\infty}\frac{\left(3n^2+2n-5\right)-\left(4n^2+5n-7\right)}{\sqrt{3n^2+2n-5}+\sqrt{4n^2+5n-7}}=$$
$$\lim_{n\to\infty}\frac{-n^2-3n+2}{\sqrt{3n^2+2n-5}+\sqrt{4n^2+5n-7}}\overset{[\frac{-\infty}{+\infty}]}{=}\left|\begin{array}{c}\text{dzielimy licznik i mianownik przez}\\ \text{"największe" wyrażenie z }\textbf{mianownika}\\ \text{(w naszym wypadku przez } n\text{ (!!!))}\end{array}\right|=\lim_{n\to\infty}\frac{-n-3+\frac{2}{n}}{\sqrt{3+\frac{2}{n}-\frac{5}{n^2}}+\sqrt{4+\frac{5}{n}-\frac{7}{n^2}}}=$$
$$\lim_{n\to\infty}\frac{-n-3}{\sqrt{3}+\sqrt{4}}\overset{\left[\frac{-\infty}{\sqrt{3}+2}\right]}{=}-\infty.$$

11. Obliczyć $\lim\limits_{n\to\infty}\left(\sqrt{3n^2+2n-5}-\sqrt{3n^2+3n-7}\right)$. Mamy

$$\lim_{n\to\infty}\left(\sqrt{3n^2+2n-5}-\sqrt{3n^2+3n-7}\right)\overset{[+\infty-(+\infty)]}{=}\left|\begin{array}{c}\text{zamieniamy różnicę na iloraz}\\ \text{za wzoru } a-b=\dfrac{a^2-b^2}{a+b}\end{array}\right|=\lim_{n\to\infty}\frac{\left(3n^2+2n-5\right)-\left(3n^2+2n-7\right)}{\sqrt{3n^2+2n-5}+\sqrt{3n^2+2n-7}}=$$
$$\lim_{n\to\infty}\frac{2}{\sqrt{3n^2+2n-5}+\sqrt{3n^2+2n-7}}\overset{[\frac{2}{+\infty}]}{=}\cos\tfrac{\pi}{2}.$$

12. Obliczyć $\lim\limits_{n\to\infty}\left(\sqrt[3]{8n^3+2n^2+n-5}-\sqrt[3]{8n^3+3n^2-7n}\right)$. Mamy

$$\lim_{n\to\infty}\left(\sqrt[3]{8n^3+2n^2+n-5}-\sqrt[3]{8n^3+3n^2-7n}\right)\overset{[+\infty-(+\infty)]}{=}\left|\begin{array}{c}\text{zamieniamy różnicę na iloraz}\\ \text{za wzoru } a-b=\dfrac{a^3-b^3}{a^2+ab+b^2}\end{array}\right|=$$
$$\lim_{n\to\infty}\frac{\left(8n^3+2n^2+n-5\right)-\left(8n^3+3n^2-7n\right)}{\left(\sqrt[3]{8n^3+2n^2+n-5}\right)^2+\sqrt[3]{8n^3+2n^2+n-5}\cdot\sqrt[3]{8n^3+3n^2-7n}+\left(\sqrt[3]{8n^3+3n^2-7n}\right)^2}=$$
$$\lim_{n\to\infty}\frac{-n^2+8n-5}{\left(\sqrt[3]{8n^3+2n^2+n-5}\right)^2+\sqrt[3]{8n^3+2n^2+n-5}\cdot\sqrt[3]{8n^3+3n^2-7n}+\left(\sqrt[3]{8n^3+3n^2-7n}\right)^2}\overset{[\frac{-\infty}{+\infty}]}{=}$$
$$\left|\begin{array}{c}\text{dzielimy licznik i mianownik przez}\\ \text{"największe" wyrażenie z }\textbf{mianownika}\\ \text{(w naszym wypadku przez } n^2\text{ (!!!))}\end{array}\right|=$$
$$\lim_{n\to\infty}\frac{-1+\frac{8}{n}-\frac{5}{n^2}}{\left(\sqrt[3]{8+\frac{2}{n}+\frac{1}{n^2}-\frac{5}{n^3}}\right)^2+\sqrt[3]{8+\frac{2}{n}+\frac{1}{n^2}-\frac{5}{n^3}}\cdot\sqrt[3]{8+\frac{3}{n}-\frac{7}{n^2}}+\left(\sqrt[3]{8+\frac{3}{n}-\frac{7}{n^2}}\right)^2}=$$
$$\lim_{n\to\infty}\frac{-1}{\left(\sqrt[3]{8}\right)^2+\sqrt[3]{8}\cdot\sqrt[3]{8}+\left(\sqrt[3]{8}\right)^2}=\tfrac{1}{6}\sin\left(2\pi-\tfrac{\pi}{6}\right).$$

W kolejnych zadaniach wykorzystamy wzór na granicę specjalną o symbolu $[1^\infty]$, czyli

$$\lim_{n\to\infty}\left(1+\frac{a}{f(n)}\right)^{f(n)} \stackrel{[1^\infty]}{=} e^a,\ a\in\mathbb{R}-\{0\}. \tag{1}$$

13. Obliczyć $\lim\limits_{n\to\infty}\left(\dfrac{5n+2}{5n-1}\right)^{5n-1}$. Mamy

$$\lim_{n\to\infty}\left(\frac{5n+2}{5n-1}\right)^{5n-1} \stackrel{[1^\infty]}{=} \lim_{n\to\infty}\left(\frac{\overbrace{5n-1+3}^{5n+2}}{5n-1}\right)^{5n-1} = \lim_{n\to\infty}\left(1+\frac{3}{5n-1}\right)^{5n-1} \stackrel{(1)}{=} e^3.$$

14.

$$\lim_{n\to\infty}\left(\frac{2n-3}{2n+5}\right)^{3n-7} \stackrel{[1^\infty]}{=} \lim_{n\to\infty}\left(\frac{\overbrace{2n+5-8}^{2n-3}}{2n+5}\right)^{3n-7} = \lim_{n\to\infty}\left(1+\frac{-8}{2n+5}\right)^{3n-7} =$$

$$\lim_{n\to\infty}\left[\left(1+\frac{-8}{2n+5}\right)^{2n+5}\right]^{\frac{3n-7}{2n+5}} \stackrel{(1)}{=} \left(e^{-8}\right)^{\lim\limits_{n\to\infty}\frac{3n-7}{2n+5}} = \left(e^{-8}\right)^{\frac{3}{2}} = e^{-12}.$$

15.

$$\lim_{n\to\infty}\left(\frac{4n+3}{4n+5}\right)^{2n^2+3n+7} \stackrel{[1^\infty]}{=} \lim_{n\to\infty}\left(\frac{4n+5-2}{4n+5}\right)^{2n^2+3n+7} = \lim_{n\to\infty}\left(1+\frac{-2}{4n+5}\right)^{2n^2+3n+7} =$$

$$\lim_{n\to\infty}\left[\left(1+\frac{-2}{4n+5}\right)^{4n+5}\right]^{\frac{2n^2+3n+7}{4n+5}} \stackrel{(1)}{=} \left(e^{-2}\right)^{\lim\limits_{n\to\infty}\frac{2n^2+3n+7}{4n+5}} \stackrel{[e^{-\infty}]}{=} 0.$$

16.

$$\lim_{n\to\infty}\left(\frac{n^2+7n+3}{n^2+4n+5}\right)^{2n^2+3n+7} \stackrel{[1^\infty]}{=} \lim_{n\to\infty}\left(\frac{n^2+4n+5+3n-2}{n^2+4n+5}\right)^{2n^2+3n+7} = \lim_{n\to\infty}\left(1+\frac{3n-2}{n^2+4n+5}\right)^{2n^2+3n+7} =$$

$$\lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2+4n+5}{3n-2}}\right)^{2n^2+3n+7} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{n^2+4n+5}{3n-2}}\right)^{\frac{n^2+4n+5}{3n-2}}\right]^{\frac{3n-2}{n^2+4n+5}\left(2n^2+3n+7\right)} \stackrel{(1)}{=} \left(e^1\right)^{\lim\limits_{n\to\infty}\frac{(3n-2)\left(2n^2+3n+7\right)}{n^2+4n+5}} \stackrel{[e^{+\infty}]}{=} +\infty.$$